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## Internetting of fires

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# NAVAL POSTGRADUATE SCHOOL Monterey, California



## Internetting of Fires

by

David Olwell and Alan Washburn

June 2002

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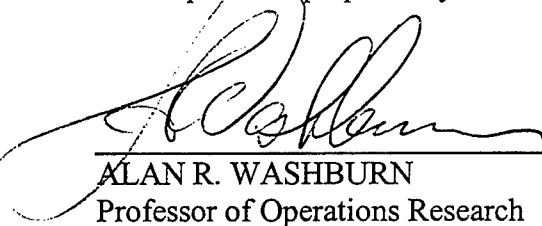
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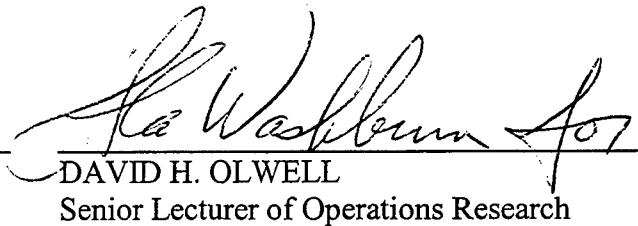
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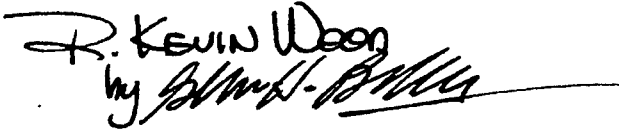


ALAN R. WASHBURN  
Professor of Operations Research




DAVID H. OLWELL  
Senior Lecturer of Operations Research

Reviewed by:




R. KEVIN WOOD  
Associate Chairman for Research  
Department of Operations Research

Released by:



JAMES N. EAGLE  
Chairman  
Department of Operations Research



DAVID W. NETZER  
Associate Provost and Dean of Research

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# Internetting of Fires

by David Olwell and Alan Washburn

## ***Abstract***

The advent of long-range, accurate weapons, together with advanced sensors and information systems, has changed the nature of the modern battlefield. Targets can now be engaged by weapon systems that are widely separated geographically from their targets. These new capabilities bring increased demands for coordination and optimization when multiple weapon systems ("shooters") engage multiple targets. There are aspects of an assignment problem where each shooter does what he is good at *relative to other shooters* in order to maximize destruction to any given set of targets. This report introduces methods for making assignments optimally, and compares them to simpler methods that optimize only locally, instead of globally. The benefits for global optimization depend on circumstances, but can be substantial.

## ***Executive Summary***

Here we introduce a simplex-type algorithm for optimally assigning shooters to targets, and compare it to other methods. The algorithm rapidly solves the problem of optimally assigning a fixed collection of shooters to a fixed collection of targets, given known kill probabilities for each potential assignment. The algorithm itself is represented by *SIMPLL.dll*, a Windows dynamic-linked library.

Optimal methods, of which SIMPLL is but one example, are compared to other methods that make decisions locally, rather than globally. These comparisons are made using *NetFire.xls*, an Excel workbook that implements all methods simultaneously so that they can be compared on specific scenarios. Optimal methods are bound to win such a comparison, but the interesting question is whether the margin of victory is big enough to justify the C4ISR system that must accompany the global point of view in practice. The answer, unfortunately, is that the margin of victory depends strongly on circumstances. In cases where all targets are equal and all shooters have the same capability, there is little to be gained by global coordination. In more variable

circumstances, the margin of victory can be large.

It is sometimes suggested that warfare can be conducted by autonomous “agents” that operate and make decisions based only on locally available information. It is also sometimes suggested that modern armed forces should attempt to achieve a C4ISR system sufficiently sophisticated to achieve “internetting-of-fires”, since it is only with such a system that the capabilities of long-range weapons can be exploited to the fullest. These suggestions are in conflict, since the one asserts that local optimization is sufficient or at least inevitable, while the other asserts that only global optimization will do. In an abstract way, these ideas are represented by the different targeting schemes discussed in this report. Possibly *NetFire.xls* and/or *SIMPLL.dll* will prove useful to those interested in exploring this issue further.

We have found instances where global use of information can achieve the same results as local use of information, but at a cost of only half the number of weapons. We have also found instances where there is no advantage to global use of information. We explain some of the factors that influence the difference.

## **1. Introduction**

Modern warfare is distinguished by the presence of increasing amounts of relevant information on the battlefield. The fog of war will never disappear, but it has thinned a bit due to continuing improvements in sensors, computers and communication networks. It is now realistic to talk about “internetting of fires” in the sense that targeting information used by a given shooter may come from sensors not under his direct control. But this internetting capability comes at a stiff price. There are even questions about whether the money might be better spent in more effective ways. General Howell M. Estes III, USSPACECOM, has put it succinctly:

*Hard choices need to be made between investments in information infrastructure [and] the combat systems themselves. This is an extreme dilemma, because combat systems, without timely, relevant information, are useless. On the other hand, you can't take out an enemy tank with just information. We need to strike a balance between 'shooters' and 'information systems' if we're going to be successful in the future.*

The subject of this report is the value of information in the context of land combat, with emphasis on the tradeoff between information and firepower. To get at this tricky issue, we postulate a group of targets, each of which has a value, and compare various targeting schemes on the basis of the Measure of Effectiveness (MOE) "fraction of total value killed, on the average". Clever schemes with a high investment in information systems can be expected to achieve high scores, but so can less advanced schemes that simply have more firepower. The incremental dollar should be spent in whatever manner increases the MOE the most. The resulting combat model, NetFire, is embodied in an Excel workbook.

The general concept in NetFire is that there is a certain length of time called a decision cycle within which no Battle Damage Assessment (BDA) is possible. Within that cycle, a fixed set of weapons ("shooters") is available to be fired at the enemy. NetFire reports results for several different methods of targeting, some methods requiring only local information and some requiring global information. One of the latter is a system wherein each shooter is directed to fire in such a manner as to maximize the MOE. The algorithm for doing this (SIMPLL) is a principal output of this research, and may be of use more generally than in NetFire.

NetFire is an abstract combat model. The enemy does not move or shoot back, and, with one exception, time is not represented. "Combat" simply consists of one-off situations where dispersed shooters must simultaneously commit to targets. Nonetheless, NetFire can be used to explore the significance of certain kinds of information, specifically BDA information and coordinating information. The next section describes the analysis that underlies the calculations. The specifics of using the Excel workbook *NetFire.xls* will be described later.

## 2. Analysis of Fire Allocation Methods

### Cases RANDOM, GREEDY, and OPT

Information may or may not be important to determining the battle outcome. Consider the following scenario. Suppose that there are  $m$  shooters and  $n$  targets, with each shooter having a single shot that is capable of killing its target with probability  $P_k$ . Temporarily make the following assumptions:

- the weapons are all “long range” in the sense that any shooter can physically attack any target;
- each target has the same value;
- each shooter possesses the required targeting information for each target; and
- there is no BDA between shots over the time period under consideration, so all shots might as well be committed in one salvo.

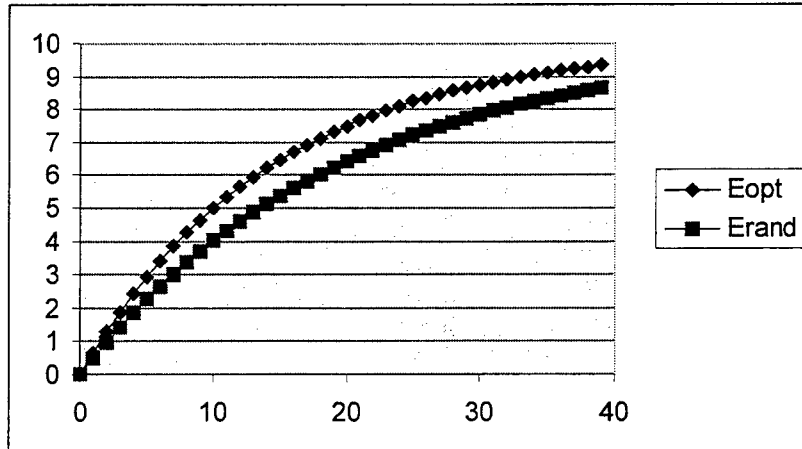
It does not follow from these assumptions that shooters can communicate with each other, nor does it follow that there is a commander capable of sensing the overall situation and giving appropriate orders. How much is this additional coordinating information worth?

The worst possible outcome from the viewpoint of the shooters would be if all  $m$  shots were directed against a single target, in which case the unfortunate target would be overkilled (unless  $P_k$  is very small), while  $n-1$  targets would not be attacked at all. The optimal outcome in this case would be if all targets were shot at equally. Clearly, achieving the best outcome requires a commander with perfect information. However, the worst outcome *also* requires a commander with perfect information, albeit with a different motivation, so comparing the best with the worst will not be enlightening. If the shooters cannot communicate—either with each other or with the commander—then it makes sense to assume that the target chosen by one shooter is statistically



independent of the targets chosen by other shooters. The resulting outcome will be intermediate in terms of outcome, but represents an information extreme in the sense that no coordination whatever is required to achieve it. It is these two extreme situations, random versus optimal, that should be compared in discussing the value of coordinating information.

The optimal allocation will allocate the shots equally, so  $m/n$  shots will be assigned to each target. Each target survives only if all of these shots miss, so the average number of targets killed will be  $E_{opt} = n(1 - (1 - Pk)^{m/n})$ . On the other hand, if each shot is directed at a random target, then each target survives a given shot with probability  $1 - Pk/n$ , independently of the other shots. Since there are  $m$  shots in total, the average number of targets killed will be  $E_{rand} = n(1 - (1 - Pk/n)^m)$ . Figure 1 shows both of these quantities as a function of  $m$  when  $n=10$  and  $Pk=0.5$ .



**Figure 1: Average number killed versus total number of shots.**

It can be seen from Figure 1 that there is very little difference between the optimal and random results when  $m$  is zero (in which case both quantities are of course 0) or very large (in which case both approach  $n$ ). Even in the middle, where  $mPk$  and  $n$  are approximately equal, the functions are surprisingly close, given the extreme differences between the command and control

systems being modeled. When all weapons are equally effective, all targets are equally valuable, and the kill probability is not too large, there turns out to be only a limited payoff for information that would permit the commander to make an optimal assignment for every weapon. When the global object is simply to distribute weapons evenly over targets, the local tactic of assigning weapons uniformly at random is not that far from global optimality.

In mixed scenarios where target values differ and the kill probability depends on both weapon and target, the situation changes. It is no longer clear what firing strategy should be used in the extreme case of no coordination, since now there is a conflict between treating all targets equally and selecting the target for which the kill probability is largest. To be more precise about this, as well as to account for the type of internetting information that makes firing even feasible, we introduce the following notation:

$i$  indexes shooters,  $k$  indexes targets

$V_k$  = value of target  $k$

$S_i$  = number of shots from shooter  $i$

$p_{ik}$  = prob that one shot of  $i$  kills  $k$ , given  $i$  shoots at  $k$

$I_{ik}$  = 1 if it is feasible for  $i$  to shoot at  $k$ , else 0 {depends on weapon range, internetting, and possibly other command and control factors}

$P_{ik} = p_{ik} I_{ik}$  {only the product of  $p$  and  $I$  is important}

When there is no commander able to coordinate fires, each shooter could still choose a target at random. But a target truly chosen at random could very well have a zero kill probability, so instead we assume that the randomization is done only over those targets for which the kill probability is nonzero, the ones that are reasonably close to the shooter. To be precise, we define

**Case RANDOM:** Assume that each shot from  $i$  is equally likely to be taken against any

target  $k$  for which  $P_{ik} > 0$ . Define the sets  $T_i \equiv \{k | P_{ik} > 0\}$  and  $R_k \equiv \{i | P_{ik} > 0\}$ , and let  $\#()$  count the number of elements in a set. Then  $P_{ik}/\#(T_i)$  is the probability that each of  $i$ 's independent shots kills target  $k$ , and therefore the probability that target  $k$  survives is  $q_k = \prod_{i \in R_k} \left(1 - \frac{P_{ik}}{\#(T_i)}\right)^{s_i}$ . The survival events are not independent among the targets, but nonetheless the expected remaining target value is  $V^* = \sum_k V_k (1 - q_k)$ .

Case RANDOM could still be criticized for sometimes directing fire against targets where the kill probability is low. It is tempting instead to assume that shooter  $i$  directs all of his fire against the target for which the product of target value and kill probability is largest. This "GREEDY" procedure is also feasible without coordination by a commander as long as target values can be sensed locally. To be precise, we define

**Case GREEDY:** Assume instead that each of  $i$ 's shots is taken against the target for which it is most effective, namely target  $K(i) \equiv \arg\max_k \{V_k P_{ik}\}$ , and let  $R_k = \{i | K(i) = k\}$  be the set of shooters who attack target  $k$ . Then  $q_k = \prod_{i \in R_k} (1 - P_{ik})^{s_i}$ , and the expression for  $V^*$  is as before.

Case GREEDY may not do as well as Case RANDOM. Suppose that there were some single valuable target that is very vulnerable to fire from every shooter. Then all fire would be directed against that one target in Case GREEDY, and all others would escape. Case GREEDY makes no provision for spreading fire over other targets.

The fact that neither RANDOM nor GREEDY is clearly the right thing to do when no coordination is possible simply bespeaks a difficult fire allocation problem. In this circumstance, one might suspect that a coordinating commander could make more of a difference, particularly when at least some of the kill probabilities are large. Our reasons for introducing the RANDOM

and GREEDY procedures are not that we advocate them in any sense, but that they are simply two procedures that might characterize what actually happens in the absence of the coordination that internetting makes possible. Specifically, they might characterize target selection by “agents” whose behavior is based only on locally sensed conditions.

The best scenario for coordination information, then—the case where its value in terms of additional targets killed is largest—will be when:

- targets have different values;
- kill probability is variable;
- at least some kill probabilities are large; and
- the scenario is not one of massive overkill or underkill.

It might seem that this is a rather narrow scenario, and in a way it is. Nonetheless, our intention is to concentrate on cases of this sort. Our contention is that this situation characterizes modern warfare. In particular, modern weapons may be guided weapons that are long range and have large kill probabilities, but which are in limited supply on account of being expensive. It is particularly important to employ such weapons carefully, reserving them for cases where less expensive weapons cannot do the job. Only coordinating information can enable this.

When coordinating information is available and timely commands to individual shooters are feasible, the fires of all the shooters can be directed in a manner that maximizes the average target value killed. The resulting optimization problem might be called a nonlinear assignment problem, since the objective will be best accomplished by assigning each shot to the target at which it is most effective. Let the variable  $x_{ik}$  represent the number of shots made by shooter  $i$  at target  $k$ . Then Case OPT is to

$$\text{maximize } \sum_k V_k(1 - q_k), \text{ subject to the constraints}$$

$$q_k = \prod_i (1 - P_{ik})^{x_{ik}},$$

$$\sum_k x_{ik} \leq S_i,$$

$$x_{ik} \geq 0$$

Here, and in all following optimization formulations, we use the convention that variable names begin with lower case letters, whereas constants begin with upper case letters. The formula for  $q_k$ , the miss probability for target  $k$ , reflects the usual independence assumption in the extended product of miss probabilities.

Case OPT is actually a nontrivial Mathematical Programming problem, since the objective function is nonlinear and the variables are also required to be integral in value. We cannot solve it in reasonable amounts of time for large problems. However, an upper bound on the optimal objective function can be obtained by relaxing the integrality constraint, which produces a simpler, continuous optimization problem for which an efficient solution technique is known (Washburn[1995]). A lower bound can then be constructed by rounding the solution to integers in such a manner that shot constraints are still enforced. This process is carried out in *SIMPLL.dll*, a dynamic-linked library that can be used with any Windows-based application. The SIMPLL algorithm is actually a branch-and-bound method (Washburn[1998]) that will find a solution accurate to within *TOL*, a nonnegative tolerance for error.

#### **Other Possible Views of the Fire Allocation Problem**

The basic viewpoint taken above is that finite stocks of weapons must be allocated to targets within any given decision cycle. The weapons themselves are not “costly” in any sense other than being in limited supply. This is a common point of view, but by no means the only reasonable one. If weapons are costly, instead of being constrained, it may make sense to minimize the cost of “getting the job done” in some sense. This new point of view results in the

consideration of optimization problems such as NLP1:

minimize  $\sum_{i,k} C_i x_{ik}$ , subject to the constraints

$$q_k = \prod_i (1 - P_{ik})^{x_{ik}},$$

$$\sum_k V_k q_k \leq V,$$

$$x_{ik} \geq 0$$

In NLP1, the weapon constraints  $S_i$  have been replaced by weapon costs  $C_i$ , and the object is to minimize the total cost of shooting, subject to the total surviving value not exceeding  $V$ . Costs may be in units of dollars, or may reflect the weight or volume of the weapons, which would be a consideration when moving weapons into a distant theater. Cost might also reflect tactical preferences.

NLP1 is a nonlinear but convex program, since the target value constraint amounts to requiring that a convex function not exceed  $V$ . NLP1 can therefore be solved by any of a number of commercially available nonlinear optimization packages. If the variables are required to be integers, a branch-and-bound algorithm could be based on an NLP1 relaxation, as in SIMPLL. However, there is no tailored algorithm for solving NLP1. We have investigated solutions to this problem using GAMS (Brooks, Kendrick, and Meeraus[1988]) to access general purpose solvers, and find that it solves much more slowly than Case OPT. For example, solving a 25 shooter, 75 target problem takes about a second for SIMPLL, but a minute for GAMS using the formulation shown in the Appendix. The performance spread increases as the problem size grows, highlighting the advantages of having a special-purpose tailored algorithm such as SIMPLL.

In principle, one could also solve NLP2

minimize  $\sum_{i,k} C_i x_{ik} + \sum_k V_k q_k$ , subject to the constraints

$$q_k = \prod_i (1 - P_{ik})^{x_{ik}},$$

$$x_{ik} \geq 0$$

NLP2 is the problem of minimizing total cost, including the “cost” of the surviving targets, subject to no constraints on either target value or weapon consumption. Weapon consumption is left to be whatever is natural for the target complex. NLP2 is a Lagrangian relaxation of Case OPT as well as of NLP1, so it should be easier to solve than either one. A special-purpose algorithm similar to SIMPLL could surely be developed for it. NLP2 suffers from the practical problem that both weapons and targets must be valued on the same scale, probably dollars. The tradeoffs that are implicit in Case OPT and NLP1 are explicit in NLP2.

One could also go in the opposite direction and add constraints to NLP1, rather than remove them. If every target is constrained to survive with only an input probability  $Q_k$ , one arrives at NLP3:

$$\text{minimize } \sum_{i,k} C_i x_{ik}, \text{ subject to the constraints}$$

$$q_k = \prod_i (1 - P_{ik})^{x_{ik}},$$

$$q_k \leq Q_k$$

$$x_{ik} \geq 0$$

NLP3 is especially simple because  $q_k$  can be eliminated, after which the constraint set is:

$$\sum_i x_{ik} \ln(1 - P_{ik}) \leq \ln(Q_k)$$

$$x_{ik} \geq 0$$

Since the objective function and all constraints are linear, this is a Linear Program. The problem here is that one must give a survival probability for each target, with no tradeoffs possible between targets. The minimized objective function might be disappointingly large if  $Q_k$  were made small for some hard-to-kill target.

## Case BDA

It was mentioned in the Introduction that NetFire does not model time, with one exception. Here we discuss that exception.

All of the previous calculations pertain to a “one-off” war that is one decision cycle in length, where all weapons are assigned without regard to the effects of other weapons. It is likely in such circumstances that some weapons will be assigned to targets that have already been killed by other weapons—a potentially significant waste if the weapons are expensive. In a multi-cycle war where BDA information about the status of targets is available, this waste could be prevented. To bound the value of such BDA information, we consider in this section a situation where there is no time pressure, and where every shot is followed by an instantaneous, perfect assessment of whether the target was killed or not. No real BDA system is this good and no war is so leisurely, but it is still of interest to see how much additional target value could be killed (by weapons not previously wasted on dead targets) in this circumstance.

The data  $P_{ik}$ ,  $S_i$ , and  $V_k$  has the same meaning as before, but many policies are now feasible that were previously not. These new policies all involve shooting a single shot at some target, assessing its effects, and then, depending on the effects, shooting another single shot at some possibly different target, and so on. Each of these policies is a very complicated object, since all eventualities must be allowed for. Each might be expressed as a decision tree. For example, one policy might begin, “Fire weapon 3 at target 6. Then follow branch A if target 6 is killed, otherwise follow branch B”. There are a great many possible such policies, but the number is finite. The problem of finding the one that maximizes the expected target value killed is therefore well defined, but the number of possible decision trees is large enough to make exhaustion an unattractive technique.

We know of no practical way of finding the optimal policy. Our goal here is merely to



bound from above the expected target value that such a policy might achieve. Toward that end, we define a collection of indicator random variables:

$Y_{ik}=1$  if target  $k$  is killed by weapon  $i$  (for each  $k$ , at most one of these can be 1),

$Y_k=1$  if target  $k$  is killed, and

$X_{ik}=1$  if weapon  $i$  is assigned to target  $k$  (for each  $i$ , at most one of these can be 1).

The firing policy will induce many correlations between these random variables, so independence assumptions among them are not appropriate, but the collection is still useful for formulating the problem of finding the optimal policy. We first note that  $Y_k = \sum_i Y_{ik}$ , and that the total value killed is  $Z = \sum_k V_k Y_k$ . The problem (call it P1) of finding the optimal policy can therefore be posed as maximizing  $z$ , the expected value of  $Z$ , subject to constraints

$$(1) \sum_k X_{ik} \leq S_i \text{ for all } i \text{ with certainty,}$$

$$(2) Y_k = \sum_i Y_{ik} \text{ for all } k \text{ with certainty,}$$

$$(3) Y_k \leq 1 \text{ for all } k \text{ with certainty,}$$

$$(4) \text{ other constraints.}$$

The other constraints in P1 include the crucial relationship between  $X_{ik}$  and  $Y_{ik}$ , the essence of the firing policy. Whatever that relationship, it must certainly hold that  $E(Y_{ik}) \leq P_{ik} E(X_{ik})$ . Now, using lower case letters for expected values of random variables, we can construct a relaxation of P1 that we name P2:

$$\text{maximize } z = \sum_k V_k y_k \text{ subject to}$$

$$(1) \sum_k x_{ik} \leq S_i \text{ for all } i,$$

$$(2) y_k \leq \sum_i x_{ik} P_{ik} \text{ for all } k, \text{ and}$$

$$(3) y_k \leq 1 \text{ for all } k.$$

P2 is a relaxation of P1, because a relationship that is true with certainty will also be true on the average, because sums and expected values can always be interchanged, because (4) implies that  $E(Y_{ik}) \leq x_{ik} P_{ik}$ , and because the rest of (4) has simply been omitted in P2. P2 is a simple Linear Program that provides an upper bound on what is achievable with any shoot-look-shoot policy. P2 has a direct interpretation where  $x_{ik}$  is the average number of weapons of type  $i$  used on target  $k$ , and  $y_k$  is the probability that target  $k$  is killed. (1) requires that the average number of weapons of type  $i$  used not exceed the number available, (2) requires that the effect of each shot not exceed  $P_{ik}$ , and (3) requires that target  $k$  be killed at most once.

This BDA upper bound is one of the computations made in *NetFire.xls*, using Excel's Solver to solve the Linear Program.

### **The AO Boundary**

Hierarchic military organizations sometimes partition the target set into parts, with each unit in the hierarchy being responsible for one of the parts. Doing so simplifies command relationships and minimizes the need for communications. It may also be beneficial in avoiding some of the overkill errors that Case GREEDY is subject to, where too many shooters choose a soft or valuable target. However, the partition can also be viewed as the addition of constraints to Case OPT, which can only have the effect of reducing the objective function. Is the reduction significant? An easy way to find out is to partition the battlefield into two halves, with the shooters and sensors in one half being ineffective in the other. Each of the cases discussed above can be applied separately in each half, and the results added. By comparing the results with the cases where the battlefield is not partitioned, one can measure the significance of the partition, as

in *NetFire.xls*.

### 3. *NetFire.xls*

#### JANUS Data

NetFire is distributed with a variety of defaults, including the entire database pertaining to (shooter, target) kill probabilities. This subsection will be of interest only if the user wishes to change those defaults, and may be skipped on first reading.

Most inputs to *NetFire.xls* are directly from the user. The main exception is a mechanism for incorporating kill probability data from the JANUS combat model. To do so, one must first obtain from the JANUS database a text file named *PKIL.csv* in the same format as sheet "PKIL". There should be a single header line followed by a separate line for each (weapon type, target type) pair, typically a long file because each shooter type may have several target types in JANUS. Each line after the first should give a sequence of kill probabilities (more precisely "hit and kill" probabilities) indexed by range and attitude relative to the shooter, as shown on sheet "PKIL". This file should be placed in the same directory as *NetFire.xls*, after which pressing the "Update tables from PKIL.csv" button on sheet "PKIL" will overwrite the data on that sheet with the new data in *PKIL.csv*.

The command button labeled "Update tables from this sheet" on sheet "PKIL" uses the data on that sheet to create sheets "QTable" and "RTable". As NetFire is distributed, the data shown on sheet "PKIL" is abstracted in subroutine NewQK() so that every (shooter, target) pair has a single miss probability that is valid out to a maximum range, after which the miss probability is 1. This subroutine then writes the miss probabilities to sheet QTable and the maximum ranges to sheet RTable, each in the form of a table with shooters on the rows and targets on the columns. Making miss probability be (say) a linearly interpolated function of range would require writing

new VBA code in subroutine NewQK().

Subroutine NewQK() also incorporates a weapon reliability parameter that is hardwired to .9. As a result, no miss probability on sheet QTable will be smaller than .1. This subroutine also initializes the number of shots available (the default for column 1 is 4 shots) and the sampling weight (the default for column 2 is set to 1 for all shooter types) of each shooter, as well as the values (row 1 is set to 10) and sampling weight (row 2 is set to 1) of each target. These values can subsequently be edited by hand, but will be renewed at the default values whenever the “Update tables from this sheet” button is pressed. The defaults can be changed by editing the VBA code for NewQK().

As distributed, *NetFire.xls* is unclassified because it uses an unclassified dataset from JANUS. More realistic, classified data would provide more realistic, classified results.

#### **The Main Sheet of *NetFire.xls***

The main sheet of *NetFire.xls* is the sheet named “pij”. On this sheet can be found a digital and graphical description of the current scenario, along with command buttons “Resample”, “ValKill”, and “MultiRep”. These buttons have the following functions:

- The battle involves three types of objects: shooters, sensors and targets. Command “Resample” randomly selects and writes the object properties to the “pij” sheet, thereby defining a scenario. It also shows the object locations in a chart. The user’s inputs here are the cells colored with a light green background: the number of each of the three types of object, the dimensions of the areas within which they are located, and, if desired, an AO boundary location. Resample also uses data from the sheets “QTable” (miss probabilities), “RTable” (max ranges), and “STable” (sensor data) to compute and write the miss probability matrices for both the AO case and the non-AO case. The user input cell “Value variance” increases the variability of the target

values, with input 0 leaving them all at the default.

- “ValKill” computes the MOE “average fraction of target value killed” for four cases and two AO possibilities, writing values to the rows and columns of the output cells with a light yellow background. One method (OPT) has two rows because both a feasible solution and an upper bound need to be shown. ValKill uses the data in the ranges named “shots”, “values”, “missprob”, and “missprobrl”. Two tables of miss probabilities are needed because results must be computed both with and without an AO. This data can be edited directly if desired, but will be overwritten when the Resample button is again pressed.
- “MultiRep” iterates the Resample/ValKill combination an input number of times, leaving only the last replication on sheet “pij”. The 10 numbers computed by ValKill are written on the sheet “MultiRep” in rows, plus one additional row that averages the results in each column. The desired number of iterations is in the green cell right above the button.

Figure 2 shows a screen shot of the main sheet, including part of the map showing object locations and two of the command buttons. The check boxes are included because sometimes it is useful to resample only parts of the scenario, but normally all will be checked. The 10 numbers shown next to the ValKill button are the main outputs.

### **Scenario Development in Resample**

The Resample command button creates each scenario by first creating objects called shooters, sensors and targets with randomly selected properties. All such objects have locations (.X and .Y) that are uniformly sampled based on the limits given by the user on sheet “pij”. These coordinates are shown on that sheet both digitally and graphically.

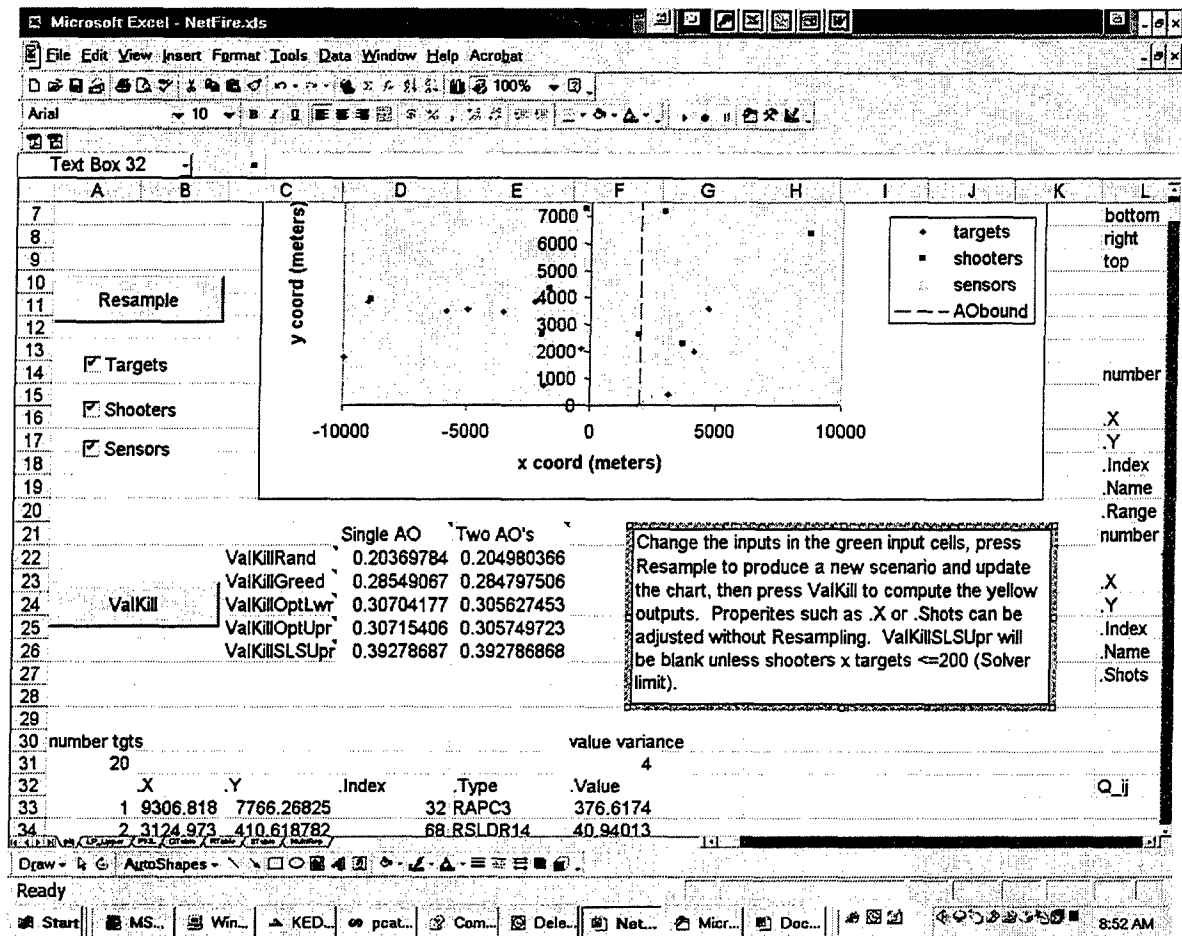


Figure 2: Screen shot of the main sheet "pij" of workbook *NetFire.xls*.

Each shooter also has a type that is selected with a sampling weight read from column 2 of sheet "QTable". These weights are all set to 1 when the sheet is created, but can be changed by the user if some shooters are desired to be more numerous than others. For example, make the weight of "PISTOL" 10 if Pistols should be 10 times as numerous as other shooters (only relative values of weights are significant, so arbitrary nonnegative values are permissible). Other editable items taken from "QTable" include:

- The number of shots available to each shooter (column 1).
- The value of each target (row 1).
- The sampling weight of each target (row 2).

- The miss probability for each (shooter, target) pair, conditional on a shot being feasible.

The maximum range for any given (shooter, target) pair is taken from sheet “RTable”.

The Resample command also creates sensors, using data on sheet “STable”. Each sensor has a name, a sampling weight, and a range within which it detects and identifies all targets. Given the user input number of sensors, the sensors are chosen randomly according to their weight, assigned the appropriate name and maximum range, and located uniformly at random within user-designated limits. As with shooter and target objects, the properties of each selected sensor are shown on sheet “pij”.

Finally, the Resample command creates two miss probability tables and shows them on sheet “pij”. The first table is located in named range “missprob”, and applies to the case where all targets are in the same area of operations (AO). The miss probability for any individual (shooter, target) pair is 1 if the shot is not feasible, or otherwise the value read from sheet “QTable”.

Feasibility requires that:

- The target must be within firing range of the shooter.
- The target must be known. A target is known if it is either
  - within range of any sensor, or
  - within VisRange of the shooter.

The VisRange parameter is hardwired at 3,000 meters in subroutine Sample(), which is the subroutine called by the Resample command to do all of the scenario generation work.

A separate table “missprobrl” is created for the AO case. All objects are partitioned by the AO boundary, and no shooter on one side of the boundary can even learn of a target on the other side, much less shoot at it. The AO miss probabilities are always larger than the non-AO miss

probabilities, since shooters and sensors in one AO are not allowed to report on or shoot at targets in the other. It does not follow that results in the AO case will always be worse (see discussion below), although they usually are.

#### **Computations Carried Out Within ValKill**

All of the inputs required by the ValKill command are displayed on sheet “pij”. In fact, the only data involved are in the named ranges “shots”, “values”, and “missprob” (non-AO) case, and “missprobrl” (AO case). All of this data can be edited directly before pressing the command button, but it will be overwritten the next time the Resample command button is pressed.

The function of the ValKill command is to compute the 10 numbers in the yellow shaded area, all of which are the MOE “average fraction of target value killed” in different circumstances. All of these computations are carried out within the ValKill() subroutine. The two local firing rules, RANDOM and GREEDY, are implemented directly within the subroutine. The OPT firing rule is implemented by calling subroutine SIMPL() in the external dynamic-linked library *SIMPLL.dll*. This subroutine returns an upper bound on what is achievable by any firing procedure faced with the same problem, so the upper bound is displayed, as well as the score achieved by the (nearly) optimal firing plan. The subroutine is declared at the beginning of module 1 of VBA code, and called from subroutine ValKill(). Finally, the BDA upper bound is calculated by calling subroutine LP\_Upr(), which formulates and solves the appropriate Linear Program using the Excel Solver and sheet “LP\_Upper”. Solver will only handle problems with 200 or fewer variables. If the product of the number of shooters and the number of targets exceeds this limit, the upper bound is simply reported to be zero. This constraint could be eliminated by replacing the standard Solver with a better optimizer.

Each of the computations is repeated in the AO case, except that miss probabilities are taken from “missprobrl” instead of “missprob”.



The `SIMPL()` subroutine in *SIMPLL.dll* can theoretically be called from any Windows application, as long as declarations are made carefully and the Compaq FORTRAN runtime library *DFORRT.dll* is also available.

NetFire was designed to generate random scenarios for analysis. It is also possible to use NetFire to analyze existing scenarios. To do so, first adjust the numbers of targets, shooters, and sensors to the correct values and press the Resample button. This will generate a random scenario with the correct dimensions. Then enter the correct target, shooter, and sensor data manually into the appropriate rows and columns. Save the workbook to protect all this work. Then uncheck the boxes for targets, shooters, and sensors under the Resample button on page “pij”. Pressing the Resample button will then calculate the two required tables of miss probabilities. The ValKill button will then evaluate the different firing strategies.

### General Tendencies

There are a few theoretical inequalities that must hold regardless of scenario. These will be stated in terms of  $VK(i,j)$ , the  $i^{th}$  row and the  $j^{th}$  column of the 10 outputs,  $i=1,...,5; j=1,2$ . Table 1 shows a typical example.

1.  $VK(i,j) \leq VK(4,j); i=1,2,3$  and  $j=1,2$ . This is because an upper bound on the optimal score must necessarily be greater than any feasible score, and all scores shown in the first three rows are feasible.
2.  $VK(i,2) \leq VK(4,1); i=1,2,3$ .  $VK(4,1)$  is an upper bound on what can be achieved by any firing method that is required to make all shots within one cycle when there is no AO boundary, and therefore is also an upper bound when the AO constraint is added.
3.  $VK(4,j) \leq VK(5,j); j=1,2$ . One basically gets to the BDA bound by replacing the function  $1-\exp(-x)$  in the OPT bound by the dominant function  $\min(1,x)$ , so the BDA

bound is necessarily at least as large as the OPT bound.

	Single AO	Two AOs
ValKillRand	0.2552852	0.245430418
ValKillGreed	0.26754648	0.245757911
ValKillOptLwr	0.31123148	0.276201159
ValKillOptUpr	0.31165106	0.276259909
ValKillSLSUpr	0.3365397	0.29697468

**Table 1: Typical outputs from *NetFire.xls*.**

All of the above inequalities should be true regardless of scenario. If any should ever fail to hold, then please notify one of the authors because a bug has been revealed.

Other than these inequalities, not much can be said in general about even the ordering of results, much less about absolute differences between them. It is not always true that GREEDY is better than RANDOM. To provide a counterexample, generate a scenario where all miss probabilities are equal. Every shooter will fire at the most valuable target in the GREEDY method, whereas the RANDOM method will spread fire equally over all. If the most valuable target is only slightly more valuable, RANDOM will get a better score. Also, in spite of inequality 2 above, it is not always bad to have two AOs. The GREEDY method, in particular, may do better when there is a constraint that prevents certain shooters from shooting at certain targets.

The incremental value of information is highly circumstance dependent, and there are so many dimensions to a NetFire scenario that one hesitates to make general conclusions beyond the above inequalities. Nonetheless, after generating and solving a large number of scenarios, any user will come to conclusions about tendencies. For the scenarios generated by the authors in the course of developing NetFire, the following tendencies have been noted:

- GREEDY generally outperforms RANDOM, but not by much.
- The OPT policy typically increases the fraction of target value killed by about 20%,

relative to either GREEDY or RANDOM.

- The BDA upper bound is typically about 30% above the GREEDY/RANDOM level.
- A few percentage points are usually lost in going from one to two AOs.

The performance of BDA is surprisingly low, since BDA as used here represents a perfect assessment system where time is not constrained and the true status of every target is always reported correctly after every shot. As a means for promoting efficiency in firing, BDA appears to be not particularly effective, at least if the scenarios examined by the authors are typical. Of course, knowing the correct status of targets has a military payoff beyond weaponizing efficiency.

All of the above observations are based on 10 shooters with four shots each. For reference, the OPT policy with two shots each is approximately as effective as GREEDY/RANDOM with four shots. In this case, optimal use of information is approximately equivalent to doubling the firing assets available. If such results survive testing in more realistic scenarios, internetting of fires will truly be worth the cost.

## Appendix

This Appendix contains the GAMS file to implement NLP1 with random scenario data. It can be amended to handle a given scenario by using input files to read the scenario data.

\$TITLE SHOOTER-TARGET ASSIGNMENT PROBLEM, MINCOST WITH  
EFFECTIVENESS THRESHOLD

\*-----DEFAULTS-----

\$OFFUPPER ONSYMLIST OFFSYMXREF

\$INLINECOM { }  
option iterlim = 10000000 ;  
option reslim = 10000000 ;  
option minlp = sbb ;  
options optcr = .4;

\*-----HEADER-----

\$ONTEXT

Original: 5/01/2002

Authors: Dave Olwell and Alan Washburn

Revised: yes, 5/09/2002

Description: Mixed Integer Non-Linear Program designed  
to find optimal assignment of shooter-target  
combinations to minimize cost( be it weight, cube,  
\$\$, or utils) while reducing the expected fraction  
of remaining target value below a given threshold.

Uses random assignment of pks, values, and costs.

Designed to explore characteristics of algorithm prior  
to developing version that imports data from files.

This is a 100 shooter, 200 target version;  
size can be adjusted by changing indices for shooters and  
target.

Sponsor: Mr. Mike Bauman, TRAC.

\$OFFTEXT

\* Option memnodes = 100000; {recommended by SBB documentation, but not allowed by  
GAMS}

\*-----INDICES-----

Sets

s shooters / sht1\*sht100 /  
t targets / tgt1\*tgt200 / ;

scalar max\_rounds maximum number of rounds fired by a shooter at a target / 3 /  
b maximum fraction remaining /.3/  
max\_v maximum target value /10/  
max\_c maximum shooter cost /8/  
good\_q initial search bound  
good\_c initial search bound  
good\_v initial search bound  
ttiv total initial target value (computed below)  
;

good\_q = .3;  
good\_c = ceil(max\_c/2);  
good\_v = ceil(max\_v/2);

\*-----PARAMETERS-----

Parameter

c(s) cost to fire a round from shts;  
c(s) = floor(uniform(1,max\_c + 1));  
{ c(s) is uniformly distributed over (1,2,3... max\_c)}

Parameter

v(t) value of target t ;  
v(t) = floor(uniform (1,max\_v + 1));  
{ v(t) is uniformly distributed over (1,2,3... max\_v)}

Parameter q(s, t) miss probabilities;

q(s,t) = max(floor(uniform(1,10))/10, floor(uniform(.5,1.5)));

{ 50% of the qs are 1 by random assignment here,  
rest are uniform over (.1, .2, .3, ..., .9)}

{ c(s), v(t), b, and q(s,t) will eventually be read in from a data file }

ttiv = sum(t, v(t));

\*-----VARIABLES -----

variables

$x(s, t)$       rounds from shti to tgtk  
 $z$               total cost of firing solution  
 $ttev$             expected remaining target value  
 $frac$              $ttev$  divided by  $ttiv$   
 $truecost$        cost without non-integer penalty;

integer variable  $x$ ;

\*\*\*\*\*Initial bounds \*\*\*\*\*

$x.up(s, t) = max\_rounds$ ;  
 {will eventually be bounded via input file ,  
 only fire MAX\_ROUNDS For a given (s, t)}

$loop((s, t), if( q(s, t) = 1, x.up(s, t) = 0))$  ;  
 {bound the solution space reasonably, can't fire at guaranteed miss}

\*\*\*\*\*Initial variable values\*\*\*\*\*

$x.l(s, t) \$(q(s, t) \leq good\_q \text{ and } c(s) \leq good\_c \text{ and } v(t) \geq good\_v) = 1$ ;  
 { pick the high value shots initially to seed search space}

\*-----EQUATIONS-----

Equations

$cost$             define objective function  
 $threshold$        $ttev$  less than  $b$  of  $ttiv$   
 $ttevdef$         defines expected remaing target value  
 $fracdef$         defines  $frac$ ;

$cost \quad .. \quad z = e = \sum( (s, t) , c(s) * x(s, t))$  ;

$threshold \quad .. \quad ttev = l = b * ttiv$  ;

$ttevdef \quad .. \quad ttev = e = \sum(t, v(t) * prod(s, q(s, t) ** (x(s, t))))$ ;

$fracdef \quad .. \quad frac = e = ttev / ttiv$ ;

model probde /all /;

```

probde.scaleopt = 1;

probde.nodlim = 50000;
* probde.optfile = 1;
probde.tryint = .25;

***** First pass -- solve relaxed problem *****
*****
solve probde using rminlp minimizing z ;
*****

***** formatted output stuff *****

display x.l, x.m ;

file output1 /output1.csv/, results /results1.txt/;

put results;
put @10, 'FORMATTED RESULTS' ///;
put 'Date of run', @15, system.date ;
put /, 'Time of run', @15, system.time;
put /, card(s), ' shooters and ', card(t), ' targets' ///;

put 'Relaxed cost', @15, z.l /;
put 'Rounded cost', @15, sum((s, t), c(s)*round(x.l(s, t))) /;
put 'Rounded effectiveness'/;
put @15, sum(t, v(t)/ttiv * prod(s, q(s, t) ** (round(x.l(s, t))))) ///;
put 'Non-zero shooter-target pairs and round count'///;

put 'Shooter', @12, 'target', @24, 'Rounds' /;
loop((s, t), if(x.l(s, t) > 0, put s.tl, @12, t.tl, @24, x.l(s, t):5:1 /)) ;
put //, 'cost', @10, z.l :6:1 ///;
put 'shooter totals' /;
loop(s, put /, s.tl, @10, sum(t, x.l(s, t)):6:1 );

put ///, 'ttiv', @25, ttiv:10:4;
put /, 'ttev', @25, ttev.l:10:4;
put ///, 'B', @25, b:4:3 ;
put /, 'fraction surviving', @25, frac.l:6:5 ;

put ///, 'List of costs';
loop(s, put /, s.tl, @10, c(s));
put//, 'List of target values';

```

```

loop(t, put /, t.tl,@10,v(t));
putclose results;

put output1; output1.pc=5; output1.pw=10000;
put 'target/shooter qs';
loop(s, put s.tl);
loop(t, put / t.tl, loop(s, put q(s, t)));
put ///;
put 'target/shooter rounds';
loop(s, put s.tl);
put /,'totals', loop(s, put sum(t,x.l(s, t)));
loop(t, put / t.tl, loop(s, put x.l(s, t)));

putclose output1;

*****

loop((s, t), x.up(s, t)$ (x.l(s, t)=0)=0);
loop( (s, t), x.l(s, t) = round(x.l(s, t)));

*****Second pass - solve integer problem with loose optcr *****
*****
solve probde using minlp minimizing z ;
*****

***** output stuff *****

display x.l, x.m ;

file output2 /output2.csv/, results2 /results2.txt/;

put results2;
put @10, 'FORMATTED RESULTS' ///;
put 'Date of run',@15,system.date ;
put /,'Time of run',@15,system.time;
put /, card(s), ' shooters and ', card(t), ' targets' /;
put 'True integer cost',@15,z.l ///;

put 'Effectiveness',@20,frac.1:5:4 ///;

put 'Non-zero shooter-target pairs and round count'///;

put 'Shooter',@12,'target',@24,'Rounds' /;

```



```

loop((s, t), if(x.l(s, t)>0, put s.tl, @12, t.tl, @24, x.l(s, t):5:1/)) ;
put //, 'cost', @10, z.l :6:1 ///;
put 'shooter totals' /;
loop(s, put /, s.tl, @10, sum(t, x.l(s, t)):6:1 );

put ///, 'ttiv', @25, ttiv:10:4;
put /, 'ttev', @25, ttev.l:10:4;
put ///, 'B', @25, b:4:3 ;
put /, 'fraction surviving', @25, frac.l:6:5 ;

put ///, 'List of costs';
loop(s, put /, s.tl, @10, c(s));
put //, 'List of target values';
loop(t, put /, t.tl, @10, v(t));
putclose results2;

put output2; output2.pc=5; output2.pw=10000;
put 'target/shooter qs';
loop(s, put s.tl);
loop(t, put / t.tl, loop(s, put q(s, t)));
put ///;
put 'target/shooter rounds';
loop(s, put s.tl);
put /, 'totals', loop(s, put sum(t, x.l(s, t)));
loop(t, put / t.tl, loop(s, put x.l(s, t)));
putclose output2;

```

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